Workshop on

PROSPECTIVE MORTALITY TABLES, LONGEVITY AND MORTALITY LINKED SECURITIES

PARIS, February, 1st, 2008

Longevity risk has become a major concern for pension and annuity business providers due to the uncertainty surrounding future mortality improvements. The aim of this workshop organized for the 20th anniversary of the Center for Research in Economic and Statistics (CREST), is to present and compare the stochastic mortality models recently introduced for constructing prospective mortality tables, or for pricing mortality and longevity derivatives. Several experts in this field will present their recent researches.

The workshop will also include a round table on: "The Development of a Liquid Market in Traded Mortality and Longevity", with discussions on the reasons explaining why longevity bonds were not offered successfully yet, the introduction of regulatory incentives, the need for a standardisation of the primitive products, the construction of transparent realized and prospective mortality indexes, the construction of ratings for mortality or longevity bonds, the links between OTC tailor made products and products tradable on bond markets.

Organizing Committee: C. Gourieroux, C. Robert, L. Taleyson

Registration form: http://www.axa.com/lib/axa/uploads/docsdd/AXA_registration_form-va.doc

CONFERENCE PROGRAM

WELCOME ADDRESS (8:45 – 9:00): **François Robinet** (CRO AXA)

SESSION 1 (9:00 – 10:15): **SMOOTHING OF MORTALITY TABLES**

Chairman: Antoine Bommier (CNRS Toulouse University)

- "*Smooth models of mortality with period shocks*", by James Kirkby (Heriot-Watt University, Edinburgh) and **Iain Currie** (Heriot-Watt University, Edinburgh)
Discussant: François Robinet (AXA Group Risk Management)
- "*P-spline projection models with international comparisons*", by **Lucie Taleyson** (AXA Group Risk Management) and Virginie Vasseur (AXA Group Risk Management)
Discussant: Jean Pinquet (University Paris X, Ecole Polytechnique)

COFFEE BREAK (10:15 – 10:45)

SESSION 2 (10:45 – 12:00): **PROSPECTIVE MORTALITY TABLES**

Chairman: Christian Hess (Paris Dauphine University)

- "*Life annuities, projected life tables, and exchangeability: An actuarial analysis in the Lee-Carter model*", by **Michel Denuit** (Catholic University of Louvain)
Discussant: Frédéric Planchet (WINTER & Associates)
- "*Quadratic stochastic intensity and prospective mortality tables*", **by Christian Gourieroux** (University of Toronto, CREST) and Alain Monfort (CNAM and CREST)
Discussant: Fulvio Pegoraro (Banque de France)

LUNCH TIME (12:00 – 13:30)

SESSION 3 (13:30 – 14:45): **STOCHASTIC MORTALITY**

Chairman: Alain Monfort (CNAM and CREST)

- "*Modelling stochastic mortality for dependent lives*", by Elisa Luciano (University of Turin, Icer and Collegio Carlo Alberto, Turin), Jaap Spreeuw (Cass Business School, London) and **Elena Vigna** (University of Turin)
Discussant: Christian Yann Robert (ENSAE, CEA)
- "*Stochastic mortality models: criteria for assessing and comparing models*", by **Andrew Cairns** (Heriot-Watt University and the Maxwell Institute, Edinburgh) David Blake, Kevin Dowd, Guy D. Coughlan, David Epstein, Alen Ong, and Igor Balevich
Discussant: Jean-Paul Laurent (ISFA, Lyon)

COFFEE BREAK (14:45 – 15:00)

PANEL SESSION (15:00 – 16:00): **THE DEVELOPMENT OF A LIQUID MARKET IN TRADED MORTALITY AND LONGEVITY**

Chairman: Pauline Barrieu (LSE, London)

Participants:
Dominic Carpenter (Director, FITCH)
Jean-Christophe Menioux (Group Treasurer, AXA)
Darryl Stewart (Vice President, JPMORGAN)
Philippe Trainar (Chief economist, SCOR)

BUSINESS BREAK (16:00 – 16:15)

SESSION 4 (16:15 – 18:15): **MORTALITY LINKED SECURITIES**

Chairman: Blaise Bourgeois (AXA Life Europe Hedging Services)

- "*On systematic mortality risk and risk-minimization with survivor swaps*", by **Mikkel Dahl** (Nordea Markets, Denmark), Martin Melchior (PFA Pension, Denmark), **Thomas Møller** (PFA Pension, Denmark)
Discussant: Alfred Galichon (Ecole Polytechnique, France)
- "*Risk and Valuation of Mortality Contingent Catastrophe Bonds*", by **Daniel Bauer** (Georgia State University, USA) and Florian W. Kramer (Ulm university, Germany)
Discussant: **Thomas Møller** (PFA Pension, Denmark)
- "*In the core of longevity risk: dependence in stochastic mortality models and cut-offs in prices of longevity swaps*", by **Stéphane Loisel** (ISFA, Lyon) and Daniel Serant (ISFA, Lyon)
Discussant: Arthur Charpentier (University Rennes I)

Abstracts:

- "Smooth models of mortality with period shocks", by James Kirkby (Heriot-Watt University, Edinburgh) and Iain Currie (Heriot-Watt University, Edinburgh)

We suppose that we have mortality data arranged in two-way tables of deaths and exposures classified by age at death and year of death. It is natural to suppose that there is a smooth underlying force of mortality, the mortality surface that varies with age and year (or period). However, observed mortality is subject to more than stochastic deviation from this smooth surface; for example, flu epidemics, hot summers or cold winters can disproportionately effect the mortality of certain age groups in particular years. We call such an effect a period shock. We describe the mortality surface with an additive model with two components: the underlying smooth surface is modelled with 2-dimensional P-splines; the period shocks are modelled with a 1-dimensional P-spline in the age direction for each year. This is a large regression model but array methods (Currie et al., 2006) enable the computations to be performed. We illustrate our methods with Swedish mortality data taken from the Human Mortality Database.

- "Life annuities, projected life tables, and exchangeability: An actuarial analysis in the Lee-Carter model", by Michel Denuit (Catholic University of Louvain)

An insurance company selling life annuities has to use projected life tables to describe the survival of policyholders. Such life tables are generated by stochastic processes governing the future path of mortality. To fix the ideas, we consider the Lee-Carter model for mortality projection. This talk purposes to examine the consequences of working with random survival probabilities. Various stochastic inequalities are derived, showing that the risk borne by the annuity provider is increased compared to the independent case. Moreover, the type of dependence existing between the insured life times is carefully examined. Several applications are discussed.

- "Quadratic stochastic intensity and prospective mortality tables", by Christian Gourieroux (University of Toronto, CREST) and Alain Monfort (CNAM and CREST)

We consider a quadratic stochastic intensity model with Gaussian autoregressive factor, derive explicit formulas for the predictive mortality tables and provide the recursive updating formulas are also provided. We also explain how to use appropriately the Kalman filter to estimate the parameters of the model and to approximate the values of the underlying factor. This methodology is applied to the French human mortality tables.

- "Modelling stochastic mortality for dependent lives", by Elisa Luciano (University of Turin, Icer and Collegio Carlo Alberto, Turin), Jaap Spreeuw (Cass Business School, London) and Elena Vigna (University of Turin)

Stochastic mortality, i.e. modelling death arrival via a jump process with stochastic intensity, is gaining increasing reputation as a way to represent mortality risk. This paper represents a first attempt to model the mortality risk of couples of individuals, according to the stochastic intensity approach. We extend to couples the Cox processes set up, namely the idea that mortality is driven by a jump process whose intensity is itself a stochastic process, proper of a particular generation within each gender. Dependence between the survival times of the members of a couple is captured by an Archimedean copula.
We also provide a methodology for fitting the joint survival function by working separately on the (analytical) copula and the (analytical) margins. First, we calibrate and select the best fit

copula according to the methodology of Wang and Wells (2000b) for censored data. Then, we provide a sample-based calibration for the intensity, using a time-homogeneous, non mean-reverting, affine process: this gives the marginal survival functions. By coupling the best fit copula with the calibrated margins we obtain a joint survival function which incorporates the stochastic nature of mortality improvements. Several measures of time dependent association can be computed out of it.
We apply the methodology to a well known insurance dataset, using a sample generation. The best fit copula turns out to be a Nelsen one, which implies not only positive dependency, but dependency increasing with age.

- "Stochastic mortality models: criteria for assessing and comparing models", by Andrew Cairns (Heriot-Watt University and the Maxwell Institute, Edinburgh) David Blake, Kevin Dowd, Guy D. Coughlan, David Epstein, Alen Ong, and Igor Balevich

We compare quantitatively eight stochastic models explaining improvements in mortality rates in England &Wales and in the US. On the basis of the Bayes Information Criterion (BIC), we find that an extension of the Cairns, Blake & Dowd (2006b) model that incorporates the cohort effect fits the England & Wales data best, while for US data, the Renshaw & Haberman (2006) extension to the Lee & Carter (1992) model that also allows for a cohort effect provides the best fit. However, we identify problems with the robustness of parameter estimates of these models over different time periods. A different extension to the Cairns, Blake & Dowd (2006b) model that allows not only for a cohort effect, but also for a quadratic age effect, while ranking below the other models in terms of the BIC, exhibits parameter stability across different time periods for both data sets. This model also shows, for both data sets, that there have been approximately linear improvements over time in mortality rates at all ages, but that the improvements have been greater at lower ages than at higher ages, and that there are significant cohort effects.

- "Stochastic mortality models: criteria for assessing and comparing models", by Andrew Cairns (Heriot-Watt University and the Maxwell Institute, Edinburgh)

We will begin by introducing a number of existing and new stochastic mortality models. We then have the task of evaluating these models in an attempt to decide which is the most suitable for a given management-decision problem. A range of criteria can be used.
At one end of the spectrum we can used the Bayes Information Criterion (BIC) to rank models. However, we will discuss a number of qualitative criteria which are all important in assessing the suitability of an individual model. In some cases, qualitative criteria call into question the appropriateness of some models that otherwise have high BIC rankings.

- "On systematic mortality risk and risk-minimization with survivor swaps", by Mikkel Dahl (Nordea Markets, Denmark), Martin Melchior (PFA Pension, Denmark), Thomas Møller (PFA Pension, Denmark)

A new market for so-called mortality derivatives is now appearing with survivor swaps (also called mortality swaps), longevity bonds and other specialized solutions. The development of these new financial instruments is triggered by the increased focus on the systematic mortality risk inherent in life insurance contracts, and their main focus is thus to allow the life insurance companies to hedge their systematic mortality risk. At the same time this new class of financial contracts is interesting from an investor's point of view since they increase the possibility for an investor to diversify the investment portfolio. The systematic mortality risk stems from the

uncertainty related to the future development of the mortality intensities. Mathematically this uncertainty is described by modelling the underlying mortality intensities via stochastic processes. We consider two different portfolios of insured lives, where the underlying mortality intensities are correlated, and study the combined financial and mortality risk inherent in a portfolio of general life insurance contracts. In order to hedge this risk we allow for investments in survivor swaps and derive risk-minimizing strategies in markets where such contracts are available. The strategies are evaluated numerically.

- “In the core of longevity risk: dependence in stochastic mortality models and cut-offs in prices of longevity swaps”, by Stéphane Loisel (ISFA, Lyon), Daniel Serant (ISFA, Lyon)

In most stochastic mortality models, either one stochastic intensity process (for example a jump-diffusion process) or a collection of independent processes is used to model the stochastic evolution of survival probabilities. We propose and calibrate a new model that takes inter-age correlations into account. The so-called stochastic logit's Deltas model is based on the study of the multivariate time series of the differences of logits of yearly mortality rates. These correlations are important and we illustrate our study on a real-life portfolio. We determine their impact on the price of a longevity swap type reinsurance contract, in which most of the financial risk is taken by a third party. The hypotheses of our model are statistically tested and various measures of risk of the present value of liabilities are found to be significantly smaller in our model than in the case of one common underlying stochastic process.